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Physica C 404 (2004) 226–229
PHYSICA C
www.elsevier.com/locate/physc

Elasticity-driven interaction between vortices in high-κ superconductors: leading role of a non-core contribution

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Abstract

We show that the elasticity-driven interaction between vortices in high-κ superconductors, for magnetic fields not so close to H_{c2}, is mainly due to a non-core contribution overlooked until now. Consequences of the accounting for this non-core contribution when discussing correlations between vortex and crystal lattices are examined.

PACS: 74.25.–q; 74.25.Qt
Keywords: Vortex interaction; Ginzburg–Landau theory

1. Introduction

The elasticity-driven interaction between vortices has turned out to be a key point, for instance, in the understanding of the observed correlations between vortex lattices (VLs) and crystal lattices (see Refs. [1–3] and the references therein). In this paper we point out that, for magnetic fields not so close to the upper critical field H_{c2} in superconductors with large values of the Ginzburg–Landau parameter κ, previous calculations of this elasticity-driven interaction between vortices notably underestimate its strength. The assumption that only the vortex cores induce strains leads to this underestimation in previous calculations [1–3]. In fact, all spatial variations of the order parameter induce strain. At first sight, core regions seem to be important sources of strain because here the order parameter varies strongly. But one has to realize that, as strain sources, the non-core regions (smooth variations) might be even more effective if their extension is large enough. It proves out that in high-κ superconductors, in which the supercurrent regions are actually very extensive, the major contribution to the vortex-induced strains is due to the non-core regions.

2. Strain-induced attraction

2.1. Non-core contribution

We will start by showing that even neglecting the vortex cores there exists a strain-induced attraction between vortices. Within Ginzburg–Landau theory the free energy per volume unit can be written as (see, e.g., Ref. [4])

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0921-4534/$ - see front matter
doi:10.1016/j.physc.2003.09.100

$$F = F_{\rm GL} + F_{\rm el} = \frac{1}{v}\int (\mathscr{F}_{\rm GL} + \mathscr{F}_{\rm el})\,dv, \qquad (1)$$

where v is the volume of the system and

$$\mathscr{F}_{\rm GL} = a|\Psi|^2 + \frac{b}{2}|\Psi|^4 + \frac{\hbar^2}{4m}\left|\left(\nabla - \frac{2ie}{\hbar c}\mathbf{A}\right)\Psi\right|^2 + \frac{H^2}{8\pi}, \qquad (2a)$$

$$\mathscr{F}_{\rm el} = r u_{ll}|\Psi|^2 + \mu\left(u_{ik} - \frac{1}{3}\delta_{ik}u_{ll}\right)^2 + \frac{K}{2}u_{ll}^2. \qquad (2b)$$

Here Ψ is the order parameter and $a = \alpha(T - T_c)$ is the only temperature dependent coefficient, T_c being the critical temperature. The elastic degrees of freedom are taken into account in Eq. (2b), where u_{ik} is the strain tensor, K and μ are the bulk and the shear modulus respectively, and summation over double indices is implied (see, e.g., Ref. [5]).

The simplest way to work out the strain-induced vortex attraction is following the method given in Ref. [6]. To minimize the free energy over the elastic degrees of freedom we distinguish between homogeneous and inhomogeneous deformations [7]:

$$u_{ij}(\mathbf{r}) = \epsilon_{ij} + \frac{i}{2}\sum_{\mathbf{k}\neq 0}[k_i u_j(\mathbf{k}) + k_j u_i(\mathbf{k})]e^{i\mathbf{k}\cdot\mathbf{r}}. \qquad (3)$$

Here, ϵ_{ij} represents the tensor of homogeneous deformations and $u_i(\mathbf{k})$ the components of the displacement vector in Fourier space. Minimization of Eq. (1) with respect to all elastic degrees of freedom gives

$$F_{\rm el} = -\frac{r^2}{2K_{4/3}}\langle|\Psi|^4\rangle - \frac{r^2}{2K}\frac{4\mu}{3K_{4/3}}\langle|\Psi|^2\rangle^2, \qquad (4)$$

where $K_{4/3} = K + 4\mu/3$ and $\langle\cdots\rangle$ means volume average.

Further minimization of Eq. (4) with respect to Ψ is not straightforward due to its non-locality. However, if $\mu = \infty$ there is another minimization procedure that avoids the treatment of non-local equations. Following Ref. [6], let us now consider this case and after that we shall return to $\mu \neq \infty$.

If $\mu = \infty$ the only possible deformation is a homogeneous dilatation u. Therefore, the free energy (1) can be written as

$$F = \frac{1}{v}\int\left[a(u)|\Psi|^2 + \frac{b}{2}|\Psi|^4 + \frac{\hbar^2}{4m}\left|\left(\nabla - \frac{2ie}{\hbar c}\mathbf{A}\right)\Psi\right|^2 + \frac{H^2}{8\pi} + \frac{K}{2}u^2\right]dv, \qquad (5)$$

where $a(u) = a + ru$, u being a variational parameter. Fixing for a while this parameter, i.e. considering for a time a clamped sample, the form of the equations of equilibrium reduces to that of the Ginzburg–Landau equations [4]. Solving them one obtains, in particular, the free energy density close to the transition between the superconducting and the mixed state in terms of the magnetic induction $B = 4\pi\mathscr{B}$. For triangular VLs in high-κ superconductors ($\ln\kappa \gg 1$) it can be written as

$$F = F_s(u) + \frac{K}{2}u^2 + \mathscr{B}[H_{c1}(u) - H] + \nu\mathscr{B}_0^{3/4}(u)\mathscr{B}^{5/4}\exp\left[-\sqrt{\mathscr{B}_0(u)/\mathscr{B}}\right], \qquad (6)$$

where $\nu = 3^{3/2}(\pi/2)^{1/2}$. The first two terms, where $F_s(u) = -a^2(u)/(2b)$, represent the free energy density in the superconducting state at $H = 0$. The third term is proportional to the vortex self-energy, where $H_{c1}(u)$ is the magnetic field at which this self-energy changes its sign. The last term represents the repulsive interaction between vortices that takes place at low flux densities. Here $\mathscr{B}_0(u) = \phi_0/[2\pi\sqrt{3}\lambda^2(u)]$ defines the reference flux density, where ϕ_0 is the flux quantum and $\lambda(u) = \{mc^2 b/[8\pi e^2|a(u)|]\}^{1/2}$ is the penetration length of the magnetic field.

When calculating H_{c1} the vortex core region can be excluded. Doing so, i.e. taking $H_{c1}(u) = \{\phi_0/[4\pi\lambda^2(u)]\}\ln\kappa$, we shall reveal the effects associated with the non-core contributions in further calculations.

Let us now return to consider free samples. Then Eq. (6) has to be minimized with respect to u. The equilibrium deformation in the superconducting state is $u_s = ar/(b\widetilde{K})$, where $\widetilde{K} = K - r^2/b$. In the mixed state there is, in addition, a deformation $u_m = u - u_s$ as a result of the creation of vortices. Since it is small close to the transition between the superconducting and the mixed states, only lowest order terms are relevant and the u_m-dependence of the repulsion term can be neglected in Eq. (6). Thus, minimizing Eq. (6) with respect to u_m we obtain

$$F \simeq \gamma F_s + \mathscr{B}(\gamma H_{c1} - H) + \nu(\gamma\mathscr{B}_0)^{3/4}\mathscr{B}^{5/4} \times \exp\left(-\sqrt{\gamma\mathscr{B}_0/\mathscr{B}}\right) - \delta\mathscr{B}^2, \tag{7}$$

where $\gamma = K/\widetilde{K}$ and $\delta = \pi[r^2/(\widetilde{K}b)]\kappa^{-2}\ln^2\kappa$ for high-κ superconductors, i.e. $\ln\kappa = 2\kappa^2 H_{c1}/H_{c2}$ [4] (here and hereafter the values H_{c1}, H_{c2}, etc. will be refereed to the non-deformed state, $u = 0$, if it is not explicitly indicated).

Having this result in mind, let us reconsider finite shear moduli. The free energy (7) could be obtained, in principle, from Eq. (4) with its coefficients corresponding to $\mu = \infty$, i.e. $r^2/(2K_{4/3}) = 0$ and $4\mu/(3K_{4/3}) = 1$. Note that there is no essential difference between the functional form of Eq. (4) for infinite and finite μ. So we conclude that the free energy density of any isotropic type-II superconductor has the form of Eq. (7) with the corresponding renormalized constants $b' = b - r^2/K_{4/3}$ and $(r^2/K)' = (r^2/K)[4\mu/(3K_{4/3})]$.

The last term in Eq. (7) represents the non-core contribution to the strain-induced attraction between vortices. Mention that this attraction disappears if the shear modulus goes to zero.

2.2. Core contribution

Let us now deduce the core contribution to the strain-induced attraction for $\mu = \infty$ in order to compare it with the non-core one. Following Ref. [2], we model the vortices as cylinders of radius ξ (coherence length) of normal phase inside a superconducting medium. Let us first consider a clamped superconducting medium. To accommodate a normal cylinder inside this medium, the cylinder should be deformed because of the difference between specific volumes of normal and superconducting phases $V_{n,s}$. Such a deformation is simply $u_0 = (V_n - V_s)/V_s$ (if $\mu = \infty$ only homogeneous deformations are possible). Let us now consider a free sample designating as n the density of cylinders (vortices). The elastic part of the free energy density can be written as

$$F_{el} \simeq n\pi\xi^2\frac{K}{2}(u - u_0)^2 + \frac{K}{2}u^2, \tag{8}$$

where u is the deformation of the sample as a whole and it has been taken into account that the bulk moduli of both normal and superconducting phases are approximately equal (K). Minimizing Eq. (8) with respect to u we obtain the equilibrium deformation of the sample: $u_m \simeq n\pi\xi^2 u_0$. Therefore, the equilibrium free energy has the contribution

$$F_{el} \simeq n\pi\xi^2\frac{Ku_0^2}{2} - n^2\pi^2\xi^4\frac{Ku_0^2}{2}. \tag{9}$$

The second term of Eq. (9) represents the attraction between vortices due to the core-induced strain. Taking into account that $n = B/\phi_0$ and $u_0 \simeq [H_c^2(T=0)/(4\pi K T_c)](\partial T_c/\partial u)$ (see e.g. Ref. [2]), where $H_c^2 = 4\pi a^2/b$ and $(\partial T_c/\partial u) = r/\alpha$, this second term in (9) can be written as

$$F_{core}^{attr} \simeq -\delta\left(\frac{H_{c2}^2\ln\kappa}{2^3\kappa^4 H_{c1}^2}\right)^2\mathscr{B}^2. \tag{10}$$

2.3. Comparison between core and non-core contributions

For high-κ superconductors one has $2\kappa^2 H_{c1}/H_{c2} = \ln\kappa$. Thus, the core contribution to the strain-induced interaction (10) and the non-core one [see Eq. (7)] are such that

$$F_{core}^{attr} \simeq \ln^{-2}\kappa F_{non\text{-}core}^{attr}. \tag{11}$$

Because in type-II superconductors the typical values of the Ginzburg–Landau parameter are $\kappa \simeq 10$–100, the core contribution to the elasticity-driven interaction between vortices is typically one or two orders of magnitude smaller than that of the non-core one, i.e. the non-core contribution is the leading one.

Working out the appropriate calculations one can see that this leading role of the non-core contribution, revealed here for elastically isotropic superconductors and magnetic fields close to H_{c1}, also takes place in the most part of the vortex state for both elastically isotropic and anisotropic superconductors [8] (see Fig. 1). Mention that when the elastic anisotropy is taken into account, there exists a dependence of the attraction energy

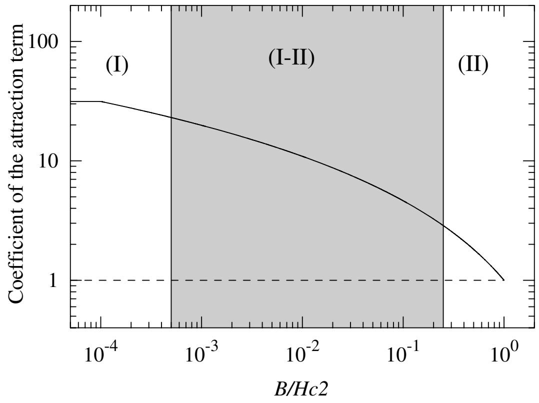

Fig. 1. Log–log plot of the coefficient of the attraction term ($\propto -B^2$) in the free energy as a function of the magnetic induction, taking into account (solid line) and neglecting (dashed line) the non-core contribution. The value of this coefficient at H_{c2}, which is the same in both cases, was taken as the unity. The indicated regions (I): $H \approx H_{c1}$, (I–II): $H_{c1} \ll H \ll H_{c2}$ and (II): $H \approx H_{c2}$, correspond to $\kappa \simeq 100$ (note that $H_{c1} \simeq 10^{-4} H_{c2}$ in this case).

on the orientation of the VL with respect to the crystal axes. This dependence, being the same for both core and non-core contribution, does not alter the ratio between them (see Ref. [8]).

3. Discussion

We have shown that previous calculations of the contribution to the VL energy due to the vortex-induced strains need essential corrections. The most important one is connected with the fact that, in high-κ superconductors, not only the vortex cores induce strains in a significant way. There also exists a significant contribution associated with the non-core regions which are, in fact, the most important ones for the VL energies at low fields ($H \ll H_{c2}$). As a result of the proper inclusion of all strain sources [8], the elasticity-driven interaction between vortices increases by a factor up to $\sim \ln^2 \kappa$ compared with the previously reported ones.

It is known since long ago that the observed correlations between VLs and crystal lattices in dirty superconductors cannot be explained without the elasticity-driven interaction between vortices [1]. This interaction has been proved to be important in clean superconductors also. For example, the VLs observed in $NbSe_2$ do not correspond to the minimum of the London energy. In Ref. [2] Kogan et al. showed that the difference in the London energies of the two possible competing structures is smaller than the difference in the energies of the corresponding elasticity-driven interactions that they calculated. Notice that Kogan et al. underestimated the elasticity-driven interaction between vortices because they assumed that only the vortex cores induce strain but, even doing so, they pointed out the importance of this interaction in $NbSe_2$. In fact this importance is even more as we have shown in the present work, what should be taken into account especially in those cases in which previous estimates concluded that the London energy was the most important one (for instance in V_3Si, see Ref. [9]).

Acknowledgements

We acknowledge L.S. Froufe for useful discussions. A.P.L. was supported from the ESF programme Vortex Matter in Superconductors at Extreme Scales and Conditions (VORTEX) and by Comunidad de Madrid (07N/005/2002). S.A.M. was supported from the Russian Fund for Fundamental Research (Grant No. 00-02-17746).

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